International Research Network Project "Symmetry, Topology and Moduli"

The 3rd International Workshop "Geometry of Submanifolds and Integrable Systems"

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Titles & Abstracts

Luis Pedro Castellanos Moscoso (Osaka City University, Japan)

Title: A classification of left-invariant symplectic structures on some Lie groups

Abstract

In geometry it is an important problem to study whether a given manifold admits some nice geometric structures. In the setting of Lie groups is natural to ask about the existence of left-invariant structures. A symplectic Lie group is a Lie group G endowed with a left-invariant symplectic form ω (i.e. a nondegenerate closed 2-form). There are many interesting results about the structure of symplectic Lie groups and considerable classification efforts in low dimensions, but the general picture is far from complete. In this talk we develop a method to classify left-invariant structures on a given Lie group: we first study the " moduli space of left-invariant nondegenerate 2-forms" and then we search inside this moduli space for the 2-forms that are symplectic. Finally we apply the method to some concrete examples.

Josef Dorfmeister (Technische Universität München, Germany)

Title: A look at integrable surface classes

Abstract

Constructing surfaces of a certain type has been one of the classical tasks of differential geometry. During the last two decades it has become increasingly clear that for certain types of surfaces, called integrable surface classes, it is efficient to apply an infinite dimensional (loop group) technique for such a construction.

In the lectures we will outline the loop group technique for a few concrete types of surfaces, but with a view at the more general case. We will make an effort to make the lectures accessible to a wide audience.

The lectures will roughly be divided up as follows:

1. Surfaces (constant mean curvature in R^3, minimal Lagrangian in CP^2 and possibly constrained Willmore), primitive harmonic Gauss maps, extended frames, and Sym type formulas
2. The loop group technique

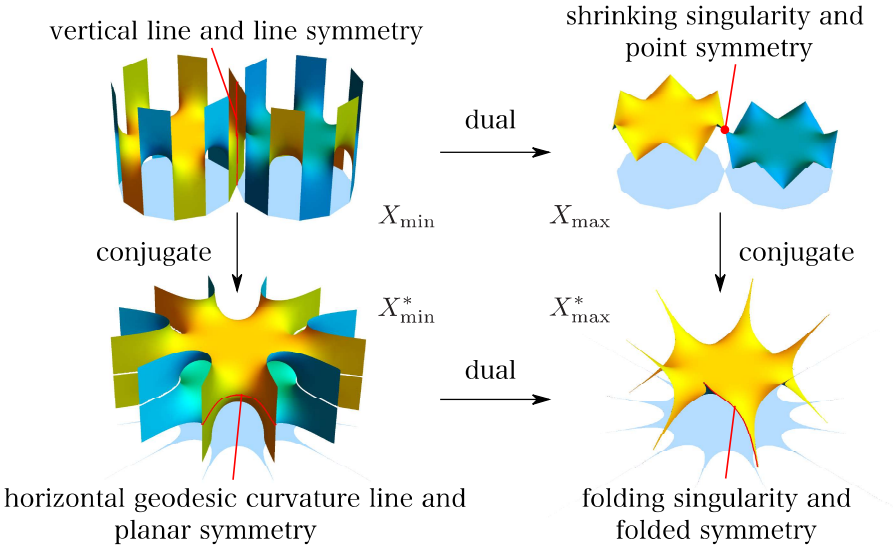
a) From extended frames to potentials and back.
b) Symmetries of surfaces and special potentials.
3. a) Special subclasses of integrable surface classes.
b) A rough classification of integrable surface classes associated with loop algebras involving $\mathfrak{sl}(2, C)$ and $\mathfrak{sl}(3, C)$.

Hiroki Fujino (Nagoya University, Japan)

Title: Duality of boundary value problems of minimal and maximal surfaces

Abstract

For a solution φ of the minimal surface equation in Euclidean 3-space over a planer domain Ω, the dual solution ψ of the maximal surface equation in Minkowski 3-space over Ω is defined. Then, there always exists a harmonic mapping f onto Ω, such that $X_{\min} = (f, \varphi \circ f)$ and $X_{\max} = (f, \psi \circ f)$ give isothermal parametrizations for graphs of φ and ψ, respectively. From this point of view, we establish a correspondence between certain boundary value problems for φ and ψ. Further, some correspondences between symmetries and singularities on $X_{\min}$ of Jenkins-Serrin type, $X_{\max}$, and their conjugations are also found. This talk is based on the joint work with S. Akamine [1].



References

[1] S. Akamine, H. Fujino, Duality of boundary value problems for minimal and maximal surfaces, arXiv:1909.00975, 2019.

Lynn Heller (Leibniz Universität Hannover, Germany)
Title: Isothermic Constrained Willmore Tori in the 3-Sphere

Abstract

In my talks I want to show that the only isothermic constrained Willmore tori with Willmore energy under 8pi are homogenous tori and 2lobed Delaunay tori. Moreover, I will compute the second variation of the Willmore Energy at these surfaces and show that the 2lobed Delaunay Tori are local minimizers under the conformal constraint.

Eduardo Mota Sánchez (University College Cork, Ireland)
Title: Unitarising the monodromy of generalised Heun's Differential Equations

Abstract

In the setup of the DPW method for constant mean curvature surfaces, we associate to the 2×2 Cauchy problem a scalar second order differential equation from the class of generalised Heun's Differential Equations. The singularities in this ODE correspond to the ends in the resulting surface. In particular, regular singularities produce asymptotically Delaunay ends while irregular singularities produce irregular ends. Our aim is to discuss global issues such as period problems and asymptotic behavior involved in the construction of constant mean curvature surfaces in R^3 arising from some generalised Heun's Differential Equations.

Thomas Raujouan (Université de Tours, France)
Title: Construction of embedded n-noids in hyperbolic space

Abstract

The DPW method induces a Weierstrass-type representation of constant mean curvature surfaces in the hyperbolic three-space. Using an implicit function theorem argument, we will show how to construct genus zero CMC $H > 1$ surfaces with any number of Delaunay ends. We will also prove that the examples having unduloidal ends are Alexandrov-embedded.

Seiichi Udagawa (Nihon University, Japan)

Title: Affine spheres and finite gap solutions of Tzitzéica equation

Abstract

1. We consider an oriented 2-manifold immersed in equi-affine 3-space. Introducing the concept of affine distance, we show that the surface considered is a proper affine sphere if and only if the surface has the property of constant affine distance. We derive the fundamental equations for indefinite proper affine sphere. We obtain the integrability condition which we must solve, which is called Tzitzéica equation.

2. We present solutions of Tzitzéica equation, denoted by (Tz), in terms of Jacobi elliptic function. We calculate the spectral curve. We rewrite the solution of (Tz) in the form of the finite gap solution.

3. We determine the Baker-Akhiezer function which represent the Blaschke immersion of indefinite proper affine sphere.

References

[1] J. Inoguchi and S. Udagawa, *Affine spheres and finite gap solutions of Tzitzéica equation*, J. Phys. Communications 2(2018) 115020, https://doi.org/10.1088/2399-6528/aaeaa().

[2] J. Inoguchi, T. Taniguchi and S. Udagawa, *Finite gap solutions for horizontal minimal surfaces of finite type in 5-sphere*, J. Integrable Systems 1(2016), pp1–34, xyw011, https://doi.org/10.1093/integr/xyw011.

Yuichi Kuno (International Christian University, Japan)

Title: The open tt^*-Toda equation and an explicit formula for Iwasawa factorization

Abstract

The open tt^*-Toda equation is a Toda-type systems of partial differential equations induced from the flatness of connections which come from combining the topological and anti-topological theory in [1]. On the other hand, from the Lie algebraic point of view, the open Toda equation is equivalent to the zero curvature condition and a reality condition. There exists an essentially one to one correspondance between the solutions of the Toda equation and holomorphic functions (called DPW data or holomorphic data). In this talk, I introduce the definition of the open tt^*-Toda equation from the Lie theoritical point of view as the analogue of [2], and give an explicit formula for Iwasawa factorization to find the solution of the open tt^*-Toda equation corresponding to given holomorphic data.

References

[1] S. CECOTTI, C. VAFA, Topological–anti-topological fusion, Nuc. Phy. B 367(1991) 359-461.

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Kazumi Tsukada (Ochanomizu University, Japan)

Title: Lagrangian submanifolds of S^6 and the associative Grassmann manifold

Abstract

It is well-known that a six-dimensional sphere S^6 admits an almost complex structure defined by its natural inclusion in the space $\mathrm{Im}\mathbb{O}$ of imaginary octonions, which is identified with seven-dimensional Euclidian space $\mathbb{R}^7$. This almost complex structure is not integrable but is nearly Kähler with respect to the induced Riemannian metric from the inner product in $\mathrm{Im}\mathbb{O}$. Many researchers have studied Lagrangian submanifolds or three-dimensional totally real submanifolds in S^6. In particular, N. Ejiri [1] proved that such submanifolds are orientable and minimal.

An oriented three-dimensional subspace of $\mathrm{Im}\mathbb{O}$ is said to be associative if it is a canonically oriented imaginary part of some quaternion subalgebra of $\mathbb{O}$. The set of all associative subspaces is called the associative Grassmann manifold, which is denoted by $\widetilde{\mathrm{Gr}}_{ass}(\mathrm{Im}\mathbb{O})$. Then it is known that $\widetilde{\mathrm{Gr}}_{ass}(\mathrm{Im}\mathbb{O})$ is an eight-dimensional compact symmetric quaternionic Kähler manifold which is identified with $G_2/SO(4)$ ([2],[7]). Here G_2 is the automorphism group of the algebra $\mathbb{O}$ of octonions.

We focus on Lagrangian submanifolds of S^6 and study the relationship of such submanifolds with the geometry of $\widetilde{\mathrm{Gr}}_{ass}(\mathrm{Im}\mathbb{O})$. Considering the Gauss maps into $\widetilde{\mathrm{Gr}}_{ass}(\mathrm{Im}\mathbb{O})$ associated to Lagrangian submanifolds, we show that those maps are harmonic. We consider the reconstruction of a Lagrangian immersion for the given Gauss map. Moreover, the Gauss maps associated to homogeneous Lagrangian submanifolds are investigated.

This is a joint work with K.Enoyoshi ([8]).

References

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Tadashi Udagawa (Waseda University, Japan)

Title: Classification of Smyth-potentials in terms of the resulting CMC-immersions, and applications

Abstract

The DPW method is a way to construct a CMC-surface, that was developed by J. Dorfmeister, F. Pedit and H. Wu for studying harmonic maps into symmmetric spaces. In the DPW method the holomorphic data determines the corresponding CMC-immersion, but this correspondence is not one-to-one and sometimes two different holomorphic data give the same CMC-immersion. In this talk, I discuss the condition that two Smyth-potentials give the same CMC-immersion and classify Smyth-potentials in terms of the resulting CMC-immersions.

References

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