

Mini-Workshop on Differential Geometry and its Discretizations

Osaka City University (Building E of Faculty of Science, Lecture Room E408)

July 3–5, 2019

Organizers

Joseph Cho (Kobe University)

Kosuke Naokawa (Hiroshima Institute of Technology)

Wayne Rossman (Kobe University)

Seong-Deog Yang (Korea University)

Masashi Yasumoto (chair, Osaka City University Advanced Mathematical Institute)

Supports

– JSPS Grant-in-Aid for Scientific Research on Innovative Areas, No.18H04489

Principal Investigator: Masashi Yasumoto, 2018-2020.

– Osaka City University Advanced Mathematical Institute

Invited Speakers

Short Lectures (3 talks)

Wai Yeung Lam (Brown University, USA)

Katrin Leschke (University of Leicester, UK)

Invited Speakers

Joseph Cho (Kobe University, Japan)

Yoshiki Jikumaru (Kyushu University, Japan)

Masahiro Kawamata (Hiroshima University, Japan)

Callum Kemp (University of Bath, UK)

Eunjoo Lee (Soongsil University, Korea)

Yuichiro Taketomi (Hiroshima University, Japan)

Program

July 3rd, 2019

13:50-14:00	Opening
14:00-14:50	Wai Yeung Lam (Brown University)
Title	Deformations of circle patterns
15:40-16:30	Yuichiro Taketomi (Hiroshima University)
Title	On a Riemannian metric whose isometry group is maximal — a special case
17:00-17:50	Katrin Leschke (University of Leicester)
Title	Isothermic surfaces via Quaternionic Holomorphic Geometry (1)

July 4th, 2019

10:00-10:50	Eunjoo Lee (Soongsil University)
Title	Characterizations of the helicoid and its associate family
11:20-12:10	Masahiro Kawamata (Hiroshima University)
Title	A construction of singular solutions for generalized Monge-Ampère equations
14:00-14:50	Yoshiki Jikumaru (Kyushu University)
Title	Variational problem for anisotropic energy and its discretization
15:40-16:30	Wai Yeung Lam (Brown University)
Title	Dimers and circle patterns
17:00-17:50	Katrin Leschke (University of Leicester)
Title	Isothermic surfaces via Quaternionic Holomorphic Geometry (2)

July 5th, 2019

10:00-10:50	Wai Yeung Lam (Brown University)
Title	Weierstrass representation of discrete minimal surfaces
11:20-12:10	Joseph Cho (Kobe University)
Title	Conserved quantities under Darboux transformations
14:00-14:50	Callum Kemp (University of Bath)
Title	Generalising the Darboux transform
15:40-16:30	Katrin Leschke (University of Leicester)
Title	Isothermic surfaces via Quaternionic Holomorphic Geometry (3)
16:30-16:35	Closing

Title and Abstract

Wai Yeung Lam (Brown University)

Title 1: Deformations of circle patterns

Abstract

A circle pattern is a realization of a graph in the plane with cyclic faces, i.e. where all vertices on a face lie on a circle. It is a central object in discrete conformal geometry. Following the ideas of William Thurston, two circle patterns with the same intersection angles are discretely conformally equivalent.

We consider the deformation space of circle patterns and introduce discrete holomorphic quadratic differentials. Their relation to the classical Teichmüller theory is discussed.

Title 2: Dimers and circle patterns

Abstract

The bipartite planar dimer model is the study of random perfect matchings (“dimer coverings”) of a bipartite planar graph. We present a correspondence between the dimer model and circle patterns, which holds for graphs that are either planar or embedded on the torus. In particular, we focus on the cluster algebra structure and the Kasteleyn matrix. The latter is related to a discretisation of the Dirac operator.

Title 3: Weierstrass representation of discrete minimal surfaces

Abstract

As an application of discrete holomorphic quadratic differentials, we present a discretisation of minimal surfaces.

Katrin Leschke (University of Leicester)

Title: Isothermic surfaces via Quaternionic Holomorphic Geometry

Abstract

In this lecture series we will introduce QHG by using the example of isothermic surfaces. Isothermic surfaces are surfaces which allow a conformal curvature line parametrisation. Well-known examples include minimal surfaces, constant mean curvature surfaces but also surfaces of revolution. We will discuss classical transformations of isothermic surfaces, such as the T-transform or the Darboux transform, in the quaternionic calculus. Additionally, we generalise the associated family of flat connections to allow for a complex parameter; in particular, we can generalise the classical transforms. In case of a CMC surface, we have an additional family of flat connections which is associated to the harmonic Gauss map of f . We discuss the link between these two integrable systems.

Joseph Cho (Kobe University)

Title: Conserved quantities under Darboux transformations

Abstract

In this talk, we look at conserved quantities of flat connections that characterize the isothermicity of a surface. In particular, we review how they behave under Darboux transformations and relate them to the discrete isothermic surface theory.

Yoshiki Jikumaru (Kyushu University)

Title: Variational problem for anisotropic energy and its discretization

Abstract

Anisotropic energy is the integral of an energy density that depends on the normal at each point over the considered hypersurface in \mathbb{R}^{n+1} . The minimizer of such an energy among all closed hypersurfaces enclosing the same $(n + 1)$ -dimensional volume is unique and it is (up to rescaling) so-called the Wulff shape. This energy is a generalization of the area and gives a mathematical model of the energy for soap bubbles or crystals.

In this talk, we will explain a recent progress of our attempt which is a theory of discrete curves and discrete surfaces based on the variational problem for anisotropic energy. As an application, we visualize the anisotropic energy gradient flow by using the software JavaView.

Callum Kemp (University of Bath)

Title: Generalising the Darboux transform

Abstract

The theory of isothermic surfaces in the conformal n -sphere may be viewed as part of a more general theory of isothermic submanifolds in symmetric R-spaces. All the usual transforms have their analogues and we will see in particular how the analogue of the Darboux transform can be characterised geometrically in terms of envelopes of certain submanifolds.

Masahiro Kawamata (Hiroshima University)

Title: A construction of singular solutions for generalized Monge-Ampère equations

Abstract

It is known that Monge-Ampère systems is a geometric formalization of Monge-Ampère equations using the theory of exterior differential systems. We introduce a generalization of Monge-Ampère systems, Monge-Ampère equations and a relationship between such systems and equations. Moreover we describe singular solutions for some generalized Monge-Ampère equations.

Eunjoo Lee (Soongsil University)

Title: Characterizations of the helicoid and its associate family

Abstract

The helicoid, along with the plane and the catenoid, is a classical example of minimal surfaces. Various characterizations have been known: a unique non-planar ruled minimal surface, a unique complete, embedded, simply-connected non-planar minimal surface, etc. We characterize a compact piece of the helicoid in a cylinder in \mathbb{R}^3 as an area-minimizing and a unique surface in certain geometrical restrictions. Furthermore, we investigate the associate family of helicoids and its uniqueness.

Yuichiro Taketomi (Hiroshima University)

Title: On a Riemannian metric whose isometry group is maximal — a special case

Abstract

In this talk, we study Riemannian metrics whose isometry groups are maximal with respect to inclusion. We get nice examples of such metrics by studying moduli spaces of left-invariant metrics on some Lie groups.